INTEGRATING ART AND MATHEMATICS IN STEAM EDUCATION: A CASE STUDY ON TILING FOR FEMALE STUDENTS

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In the era of Society 5.0—a future societal vision proposed by the Japanese government—there is an increasing demand for STEAM (Science, Technology, Engineering, Arts, and Mathematics) professionals capable of approaching problems from diverse perspectives and solving them creatively. Society 5.0 envisions the National Institute of Technology (KOSEN) as a hub for STEAM education for elementary and junior high school students, contributing to the development of a comprehensive educational framework.

Since 2019, we have incorporated STEAM education into the "Liberal Arts Seminar," a course for fourth-year students in the main academic program. In these seminars, faculty members from the humanities introduce topics such as mathematics, debate, and economics based on their areas of expertise. This interdisciplinary approach fosters collaborative learning among students from different departments, promoting the integration of knowledge and creativity. However, opportunities for students to apply their mathematical knowledge in real-world, socially meaningful contexts remain limited.

To address this gap, we developed teaching materials that integrate art and mathematics, aiming to promote early STEAM education and encourage students' societal engagement. These materials were implemented in a 2024 open course specifically designed for female junior high and high school students. The teaching assistants (TAs), all female students who had previously studied the relationship between art and mathematics in the Liberal Arts Seminar, played a vital role in facilitating the course.

The course targeted female students due to the low enrollment of women in science and engineering fields in Japan. By fostering interest in these subjects through art-integrated mathematical activities, we hope to encourage more female students to pursue STEM pathways.

The initiative received positive feedback from participants. Moreover, female TAs gained valuable experience applying their academic knowledge in educational settings. Since 2023, the course has explored the tiling of a plane using nets of regular polygons and polyhedra. In 2024, we enhanced the curriculum by incorporating more design elements, such as Escher's artworks and Penrose tilings, to emphasize the fusion of art and mathematics within a STEAM framework.

This paper presents the development and implementation of these teaching materials and evaluates their educational effectiveness through exercise results and pre- and post-course questionnaires.

Keywords: STEAM Education, Art and Mathematics, Tiling, Female Students, Teaching Assistants

Introduction

STEAM education, introduced by G. Yakman, is a pedagogical approach that uses Science, Technology, Engineering, Arts, and Mathematics as interdisciplinary entry points to guide students' inquiry, dialogue, and critical thinking for solving real-world problems.

We have explored mathematics education from a STEAM perspective and implemented it in various settings (e.g., Sakai & Tanaka, 2014; Kawashima, Sakai & Matsuda, 2021). In particular, we have conducted multiple open courses for junior high school students focused on mathematical concepts (e.g., Sakai & Miyaji, 2013; Sakai, Miyaji & Nakabo, 2013; Kawashima et al., 2018; Sakai, Nakamura & Miki, 2024).

Tiling has been a mathematical interest since ancient Greek times and can be observed in everyday settings such as floor tiles and wallpaper. The works of M.C. Escher, who illustrated human and animal figures in tessellated patterns, make tiling a highly visual and engaging subject. For this reason, it is particularly well-suited for STEAM education, as it bridges abstract mathematical theory with artistic creativity.

In 2023, we developed an open class that examined plane tiling and extended the discussion to spherical tiling. To engage female students, we created educational materials that emphasized symmetry, inspired by Escher's works.

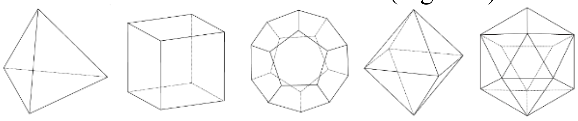
The purpose of this study is to promote early STEAM education through the development and implementation of tiling-related teaching materials, and to empower students from the Liberal Arts Seminar to serve as TAs in junior high school courses.

This paper is structured as follows: First, we define key concepts and theorems related to regular polyhedra, their net developments, and tilings. Next, we describe the structure and implementation of the open course, followed by an analysis of questionnaire results. Finally, we discuss the effectiveness of the course and outline directions for future research.

Regular polyhedrons and tiling

First, we define the regular polyhedra and provide some examples.

Definition 1. Being a regular polyhedron implies that the faces are congruent regular polygons, and the same number of faces meet at each vertex (Figure 1).



[Figure 1: Examples of regular polyhedra]

Then, the regular polyhedra are known to have the following property.

Theorem 1. Only five regular polyhedral are present: tetrahedron, hexahedron, octahedron, dodecahedron, and icosahedron, as shown in Figure 1.

The all polyhedra are known to have the following property.

Theorem 2 (Euler's polyhedron theorem). Let v, e, f be the vertices, edges, and faces of the polyhedron, respectively. Then $v - e + f = 2$ holds.

Next, we define the tiling of a plane and provide some examples.

Definition 2. When a plane is filled with the same figure without gaps or overlaps, the result is called the tiling of the plane (Figure 2).

[Figure 2: examples of tiling of a plane]

The tiling problem has been known since ancient Greek times, and it is called the "*tiling problem.*"
The tiling problem. Laying out a plane with no gaps or overlaps using several different figures is called the tiling problem.

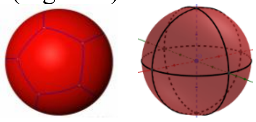
Then, the tiling of a plane is known to have the following properties.

Theorem 3. Regular polygons that tile planes are only triangles, quadrilaterals, and hexagons.
The tiling of a sphere can be defined in the same way as tiling of a plane as follows.

Definition 3. When a sphere is filled with the same (curved) figure without gaps or overlaps, the result is called the tiling of a sphere.

The tiling of a sphere is known to have the following property.

Theorem 4. A sphere can be tiled using curved regular polygons made from each face of a regular polyhedron, namely, a sphere has five types of tiling with curved regular polygons (Figure 3).



[Figure 3: examples of tiling of a sphere by 12 curved regular pentagons and 8 curved regular triangles]

Construction of open course

In this class, we set up tiling as a theme for STEAM education that integrates art and mathematics. Through activities in which participants think about the tiling of a plane and a sphere with the support of a teacher and TAs, we aim to encourage TAs to improve their teaching skills and deepen their understanding, stimulate participants' interest in mathematics, and provide them with clues to have multifaceted ideas. The purpose of the TAs is to give back to society through the acquired mathematics ability and improve teaching skills and deepen their understanding of mathematics. The remainder of this paper is organized as follows. First, as an introduction to tiling, we introduce tiling and its history through Escher's works. After defining regular polygons, we explained what kind of regular polygons can tile a plane through exercises. Finally, after defining regular polyhedra, the tiling of a sphere using curved regular polygons made from each face of a regular polyhedron will be explained through exercises.

A questionnaire was administered before and after the class to investigate changes in participants' awareness. The flow of this class is as follows:

(Pre-questionnaire)
(1) Definition and history of tiling
(2) Tiling of a plane using regular polygon
(3) Problem exercises
(4) Tiling of a sphere using curved regular polygon
(5) Problem exercises
(6) Tiling and Symmetry
(7) Problem exercises
(Questionnaire administered afterwards)

Content of our practice

In this class, we set the tiling of a plane or a sphere using (curved) regular polygon as the theme for early STEAM education and provided an opportunity for students who have studied tiling at Liberal Arts Seminar to teach them as TAs, aiming not only to arouse participants' interest in mathematics but also to provide them with clues to have multiple perspectives. This class is organized as follows:

(a) Learning contents
Tiling of a plane or a sphere using (curved) regular polygon, and Symmetry.
(b) Construction
Date: December 7, 2024
Participants: 21 elementary school students
Leaders: Two teachers and three TAs studied tiling at a Liberal Arts Seminar.
Time: 150 (90+60) minutes

At the start of the session, a questionnaire was distributed, based on the competency framework proposed by the National Institute of Technology (KOSEN) STEAM Project Working Group. The items evaluated the following competencies:

(Problem-Solving Skills)
- When you didn't understand something, did you try to look it up by yourself?
- When you faced a problem, did you try to solve it on your own?

(Critical Thinking)
- When you encountered something you didn’t understand or found strange, did you think, "Why is that?"
- When listening to your friends or teachers, did you ever think, "Why do they think that way?"

(Creativity)
- Did you enjoy coming up with fun ideas or thinking of something new?
- When doing something, did you try to add your own creative twist or personal touch?

(Flexibility)
- When plans or situations changed suddenly, were you able to stay calm and respond appropriately?
- When things didn’t go as expected, did you try various approaches to solve the problem?

(Perspective-Taking)
- When a problem occurred, did you try to consider the cause and solution from a broad point of view?

(Execution Ability)
- Did you follow through with the plans you made?
- Even when things didn’t go well, did you make efforts to complete the task to the end?

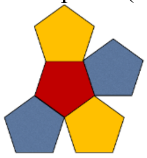
Through the lesson, we showed slides to the participants and provided explanations, as well as hints to help them solve the exercises. Participants took notes on their answer sheets before solving the problems while the TAs walked around and answered their questions. Through these activities, the TAs developed their abilities to teach others (Figure 4).

[Figure 4: the scene of open course]

In the beginning of the class, we introduced Escher's artwork and asked the participants to think about what type of figures are hidden in the paint (Figure 5).

[Figure 5: Regular Division of Plane III and IV by Escher]

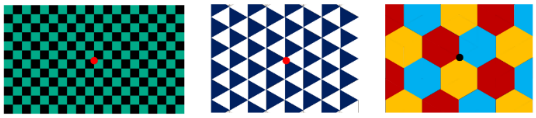
Next, we provided the definition of the tiling of a plane and its problem and property (Definition 1, tiling problem, and Theorem 2). After asking the participants if they could tile a plane with figures other than regular triangles, squares, and hexagons, we provided them an example of failure to tile a plane (Figure 6).



[Figure 6: An example of failure to tile a plane]

As one of the preparations for discussing the difference between being able to tile a plane and not, we described a pair of shapes around a vertex and their number of pieces (Figure 7).

Preparation 1. Let us consider the tiling by the following regular polygons as below. Since there are four squares around each vertex in the left figure, we denote them as (4,4). Similarly, there are six regular triangles and three regular hexagon around each vertex in the middle and the right figure, we denote them as (3,6) and (6,3), respectively (Figure 7).

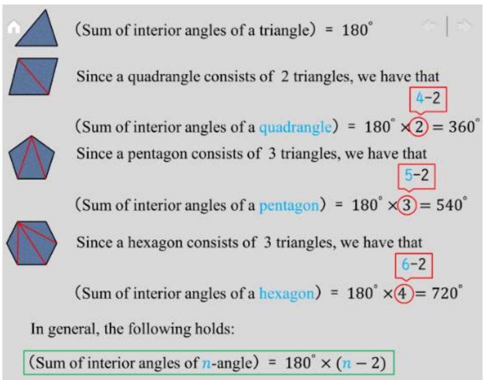


(4, 4) (3, 6) (6, 3)

[Figure 7: Examples of a pair of shapes around a vertex and their number of pieces]

In addition, as another preparation, we explained the sum of the interior angles of a polygon using the following slide (Figure 8):

Preparation 2.



[Figure 8: Sum of interior angles of a polygon]

After these preparations, we tasked the participants with the following exercises. In particular, Exercise 2 leads to the properties of regular polygons that can tile a plane.

Exercise 1. Fill in the underlined blanks below.
The sum of the interior angles of an n-angle is ___. Since all the interior angles of a regular n-angle are equal in size, the size of one interior angle is ___. Suppose that we have a plane tiled with regular n-angles. If there are p n-angles around each vertex, i.e., (n, p)-type tiling, then we have $\frac{n-2}{n} \times 180° \times p =$ ___. Dividing both of these sides by 180°, we get $\frac{n-2}{n} \times p = 2$.

Exercise 2. Fill in the underlined blanks below.
Dividing both sides of $\frac{n-2}{n} \times p = 2$ by p, we have $\frac{n-2}{n} = \frac{2}{p}$. Since left-hand side can be transformed to $\frac{n-2}{n} = \frac{n}{n} - \frac{2}{n} =$___, we have $1 - \frac{2}{n} = \frac{2}{p}$. This equation is transformed to obtain $1 = \frac{2}{n} + \frac{2}{p}$. Dividing both of these sides by 2, we have ___.

Exercises 2 and 3 can be summarized as follows:

Theorem 5. Suppose there is a plane tiled by regular n-angles. Let p be the number of regular n-angles around each vertex of a regular n-angle, then the equality: $\frac{1}{n} + \frac{1}{p} = \frac{1}{2}$ holds.

In Exercise 3, participants were asked to verify that tiling of a plane by squares, triangles, and hexagons satisfies the condition that obtained in Exercise 2. Moreover, they were also asked to verify that a regular pentagon does not satisfy this equality (Table 1).

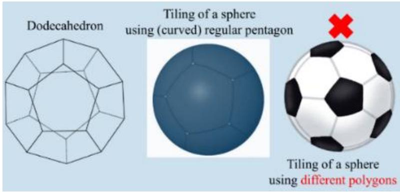
Exercise 3. Answer the following questions.
(1) Perform the calculation $\frac{1}{n} + \frac{1}{p}$ using the following table: for a regular square, a regular triangle, and a regular hexagon, where n is the number of sides of the regular polygon and p is the number of figures around the regular polygon.
(2) Consider the reason why regular pentagonal tiling is not possible.

[Table 1: Calculation of p and $\frac{1}{n} + \frac{1}{p}$]

n	p	$\frac{1}{n} + \frac{1}{p}$
3		
4		
5		
6		

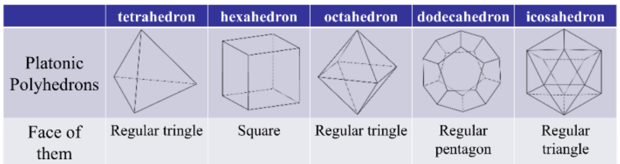
Remark 1. Suppose that there is a plane tiled by regular pentagons, then the equality: $\frac{1}{5} + \frac{1}{p} = \frac{1}{2}$, namely, $p = \frac{3}{10}$ holds by Theorem 4. This is inconsistent with p being a natural number. When n is 7 or more regular polygons, substituting condition $\frac{1}{n} + \frac{1}{p} = \frac{1}{2}$, we see that the same contradiction occurs as when $n = 5$. Thus, we have Theorem 1.

Next, we explained the tiling of a sphere using the following slide as follows: If you think of a dodecahedron made of rubber, and fill it with air like a balloon, it becomes a tiling of sphere by curved regular pentagon. The soccer ball is an example of sphere tiling, but since it uses different polygons, we will not consider it in this class.



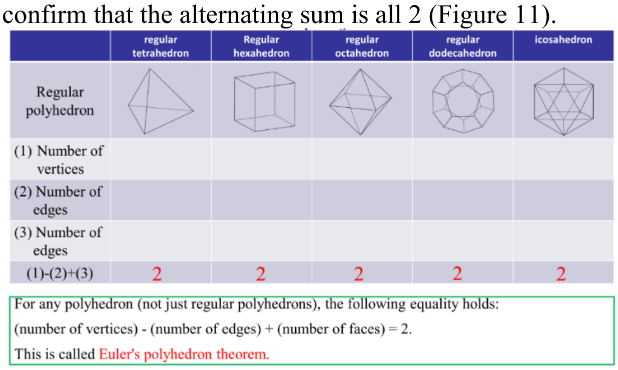
[Figure 9: Examples of tiling of a sphere and tiling of a sphere not covered in this class]

After explaining the tiling of a sphere, we asked whether there are any regular polygons, other than the (curved) regular pentagon, that can tile a sphere. Then, we introduced the regular polyhedrons called Platonic Polyhedrons and helped the participants understand that, just like in the case of the regular dodecahedron, if these polyhedrons are transformed to a sphere, each of their regular polygonal (curved) faces makes a tile of a sphere (Figure 10).



[Figure 10: Platonic polyhedrons and their faces]

To experience a property of polyhedrons called Euler's Polyhedron Theorem, we asked participants to calculate the number of vertices, number of edges, and number of faces of each of the following five polyhedrons, and to confirm that the alternating sum is all 2 (Figure 11).



[Figure 11: Euler's polyhedron theorem]

Next, the following problem was assigned to find general expressions for the number of vertices, edges, and faces of all Platonic polyhedrons.

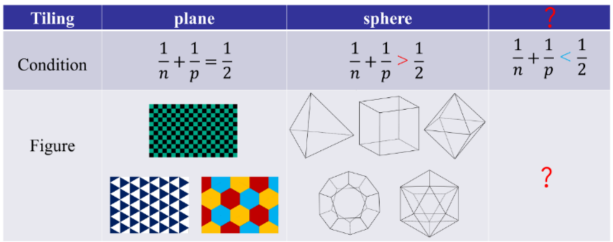
Exercise 4. Let k, n and p present the number of faces of a regular polyhedron, the number of sides of each face, and the number of faces around each vertex, respectively. Then, answer the following questions.
(1) Find n and p for all platonic polyhedrons.

(2) Represent the number of vertices and edges of all platonic polyhedrons by k, n and p, respectively.

By using Exercise 3 and Euler's polyhedron theorem, we can see that the following theorem holds.

Theorem 6. The condition for laying out a sphere with one type of (curved) regular polygon is $\frac{1}{n}+\frac{1}{p}>\frac{1}{2}$.

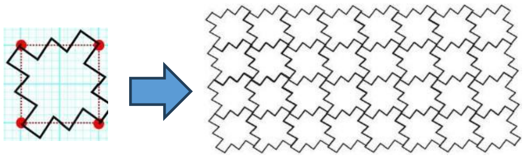
After Exercise 4, we introduced that there are only five pairs of natural numbers (n, p) satisfying $\frac{1}{n}+\frac{1}{p}>\frac{1}{2}$: (3,3), (4,3), (3,4), (5,3), (3,5). In addition, we gave the participants a summary of the class and presented one of the future tasks. In particular, I asked them what figure satisfies condition $\frac{1}{n}+\frac{1}{p}<\frac{1}{2}$, and gave them an opportunity to study tiling in the future (Figure 12).



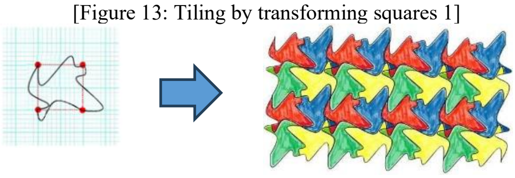
[Figure 12: Summary and future tasks]

Tiling and symmetry

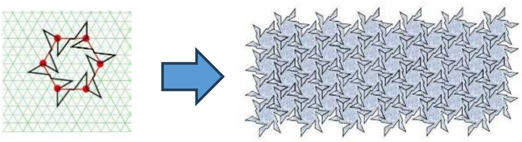
In the latter part of the class, the participants challenged themselves to tile the plane by transforming equilateral triangle, square and regular hexagon according to a certain rule (Figure 13-16).



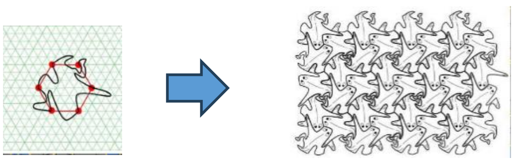
[Figure 13: Tiling by transforming squares 1]



[Figure 14: Tiling by transforming squares 2]



[Figure 15: Tiling by transforming regular hexagons 1]



[Figure 16: Tiling by transforming regular hexagons 2]

Considerations from the questionnaire results

The primary aim of this course was to deepen TAs' teaching abilities and mathematical understanding by involving them in the instruction of tiling using polyhedral developments. Simultaneously, the course aimed to cultivate participants' interest in mathematics and foster the development of multifaceted thinking.

To evaluate the effectiveness of the course, we analyzed both the accuracy of exercise responses and the results of pre- and post-course questionnaires. Each questionnaire item was scored on a 4-point Likert scale:

- **4**: Agree
- **3**: Somewhat agree
- **2**: Somewhat disagree
- **1**: Disagree

The mean scores for each competency category were calculated and compared, as shown in Table 2.

[Table 2: Average score for question item choices]

Competency	Question items	Before	After	Increase
Problem-Solving Skills	When you didn't understand something, did you try to look it up by yourself?	3.38	3.79	0.40
	When you faced a problem, did you try to solve it on your own?	3.24	3.64	0.40
Critical Thinking	When you encountered something you didn't understand or found strange, did you think, "Why is that?"	3.57	3.86	0.29
	When listening to your friends or teachers, did you ever think, "Why do they think that way?"	3.48	3.79	0.31
Creativity	Did you enjoy coming up with fun ideas or thinking of something new?	3.67	3.86	0.19
	When doing something, did you try to add your own creative twist or personal touch?	3.24	3.64	0.40
Flexibility	When plans or situations changed suddenly, were you able to stay calm and respond appropriately?	3.05	3.36	0.31
	When things didn't go as expected, did you try various approaches to solve the problem?	3.19	3.57	0.38
Perspective-Taking	When a problem occurred, did you try to consider the cause and solution from a broad point of view?	2.95	3.50	0.55
Execution Ability	Did you follow through with the plans you made?	3.05	3.43	0.38
	Even when things didn't go well, did you make efforts to complete the task to the end?	3.38	3.71	0.33

The analysis results for each competency are as follows. Problem-Solving Ability improved steadily, indicating that students became more proactive in tackling challenges independently. The activities encouraged initiative and persistence. Critical Thinking Ability showed moderate improvement. Students were already relatively strong in this area, and the workshop helped them deepen their habit of asking "why" and considering different perspectives. Creativity also saw a moderate increase. Participants enjoyed coming up with ideas and applying their own twists, showing that the environment supported imaginative thinking, though there may be room to further boost divergent thinking. Flexibility improved as students became better at adapting to unexpected changes and trying alternative approaches. The hands-on, problem-based format likely fostered this growth in resilience. Perspective-Taking showed the most significant improvement. Students learned to consider broader contexts and analyze problems from multiple viewpoints. This indicates a meaningful shift toward holistic and metacognitive thinking. Execution Ability improved as students demonstrated stronger follow-through and problem-solving persistence. They were better able to execute plans and adjust when things didn't go as expected.

The overall insights are as follows.

- The biggest growth was in *systems thinking*, suggesting that the workshop was particularly effective in helping students see the bigger picture.
- Improvements in *problem-solving* and *implementation* show strengthened self-direction and perseverance.
- While *critical thinking* and *creativity* were already relatively high, the workshop helped sustain and slightly enhance these competencies.
- The growth in *flexibility* reflects a positive impact on emotional regulation and strategic adaptability.

This competency growth profile suggests that the STEAM workshop offered a well-rounded, effective learning experience that helped students develop both cognitive and practical 21st-century skills.

Next, participants' comments were listed as evidence to verify the achievement of the objectives. The results of the comments for the participants are as follows.

- "I'm glad I was able to help and cooperate with the participants sitting nearby".
- "I found it interesting to learn why regular pentagons cannot be used for planar tiling".
- "I didn't know there was mathematics in art, so it was very intriguing".
- "It seemed simple at first, but it was actually deep, and it broadened my view of mathematics. It was a very meaningful time".
- "I was able to understand various shapes used in tiling, and regular tiling of the plane was interesting."
- "I'm glad I got to learn the characteristics of different shapes. It was the first time I realized that the planar compositions I learned in art are connected to science and mathematics".

The comments and their analyses indicate that the course has achieved some degree of success in arousing interest in mathematics. In addition, we observed the activities of the TAs and participants in class. During the exercise-solving time, we observed participants eagerly solving exercises and TAs actively approaching them and answering questions.

Conclusion of this effort and a future subject

Our goal is to "provide our students with the opportunity to teach what they have learned about tiling," and to "spark participants' interest in mathematics and give them the opportunity to gain multiple perspectives" through this course.

Comments with participants, survey responses, and observations indicated that the course is somewhat effective for learners and has potential as a STEAM resource. However, we found that the participants did not fully understand the relevance of mathematics to other fields or acquired multiple perspectives. Moreover, because the participants were interested in science, technology, and mathematics, the survey was not sufficient to determine changes in their attitudes.

In this class, we focused on tiling of a plane, but in the next class, we would like to prepare materials for proofs of tiling of a sphere and for tiling figures that satisfy condition $\frac{1}{n}+\frac{1}{p}<\frac{1}{2}$, called hyperbolic planes, which we left to the participants as a future task. The hyperbolic plane, which appeared as a plane to develop "non-Euclidean geometry," can be drawn using software called GeoGebra, and is suitable not only as a mathematics teaching material but also as a learning material for junior high school students using ICT.

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