Education Program of Radio-Frequency Impedance Matching for Kosen Grade System

Susumu Takabayashi*, Takashi Yoshitomi, Yukimura Yamasaki, Takumi Shimoda and Yamato Ikiyama

Department of Creative Engineering, National Institute of Technology, Ariake College, Omuta, Fukuoka, Japan.

S. Takabayashi* (stak@ariake-nct.ac.jp)

Impedance matching in electrical engineering is a very important concept for radio-frequency electrical circuit design. Electrical circuits without impedance matching are not power efficient and do not lead effective telecommunication. Incomplete impedance matching may cause breakdown or fire by power reflection. However, impedance matching is difficult to learn because it requires deep understanding of both the electrical circuit and electromagnetism. Here we present a step-by-step education program of impedance matching performed in National Institute of Technology, Ariake College (Ariake Kosen). This education program is ranged over three grades in the Kosen system so that students learn it without difficulty. The program comprises four practices. The education starts in the fourth grade from electromagnetism theory with Maxwell's equations including the skin effect and electromagnetic wave. Actual impedance matching of an audio circuit in the KHz region is performed in the next fifth grade. Finally, in the sixth grade (first grade of the advanced course), impedance matching of the circuit in the MHz region and that of a waveguide in the GHz region are experienced with the S-parameter analysis using a vector network analyzer (VNA). The VNA is one of the most expensive apparatuses in electrical engineering. Fortunately, a low-cost VNA such as NanoVNA is released recently. Through this program, Kosen students will be expected as radio-frequency electrical engineers.

Keywords: *Impedance matching, audio circuit, waveguide, S-parameters, vector network analyzer (VNA)*

Introduction

A voltage signal is a kind of electromagnetic wave. The wave is characterized by wavelength λ, expressed by $\lambda = {}^{c}/_{f}$ [m], where f is the frequency $\left[\text{Hz} = {}^{1}/_{\text{s}}\right]$ and c is the speed of light, 299,792,458 ${}^{\text{m}}/_{\text{s}}$ in vacuum. The wavelengths of commercial 50/60 Hz voltages are calculated as approximately 6000/5000 km. In general, people do not feel the influence of the wavelengths of voltage signals on electrical circuits.

On the other hand, the characteristics of an electrical circuit working at a radio frequency depend on its size. The wavelength of a 2.45-GHz signal, a frequency for the wireless local area network (LAN) and microwave ovens, is calculated as only 122 mm that is comparable to the actual circuit size. Figure 1 shows variation of voltage signals that are expressed by $A = A_0 \exp(j\omega t)$, where ω is angular frequency, on a 30-mm distance line without any elements on the circuit. Signals of 50/60 Hz keep constant anywhere on the line. However, another signal at a frequency of 2.45 GHz varies. Even if 0 V at the point X, the maximum signal is observed at the point Y that is ${}^{1}/_{4}\,\lambda$ apart from X. An invisible passive element must be considered.

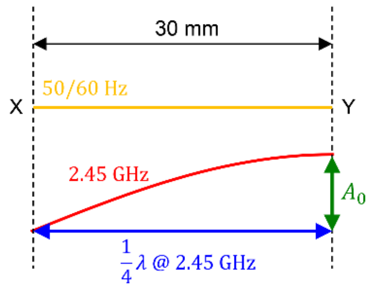

Figure 1. Voltage variation in a 30-mm distance line without any elements between X and Y. 50/60-Hz or low-frequency signals do not vary on the line; however, because the distance matches ${}^{1}/_{4}\,\lambda$ of a 2.45-GHz signal, the maximum value A_0 is observed at the point Y.

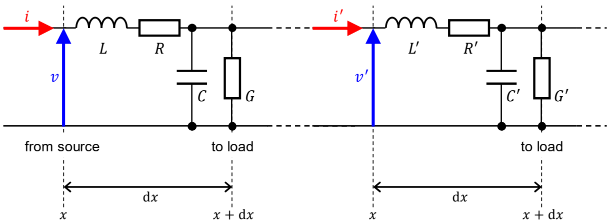

Figure 2. Sequential connection of DCCs.

When designing a radio-frequency circuit, the distributed constant circuit (DCC) analysis must be considered, instead of the lumped constant circuit (LCC) analysis that expresses a general electrical circuit. Figure 2 shows that a DCC is composed of a series of an inductor L, a resistor R, a parallel of a capacitor C, and another resistor G. The voltage and current are both a function of distance x. An actual radio-frequency circuit is regarded as sequential connection of DCCs.

A DCC is analyzed by the following telegraph equations (Equations 1 and 2) on voltage v and current i,

$$\begin{cases} -\dfrac{\mathrm{d}v}{\mathrm{d}x} = (j\omega L + R)i = Zi, & (1) \\ -\dfrac{\mathrm{d}i}{\mathrm{d}x} = (j\omega C + G)v = Yv, & (2) \end{cases}$$

where $Z \equiv j\omega L + R$ and $Y \equiv j\omega C + G$. From these equations, we obtain the voltage and current results (Equations 3 and 4),

$$\therefore \begin{cases} v = \begin{Bmatrix} A\exp(-j\omega\sqrt{LC}x) \\ +B\exp(j\omega\sqrt{LC}x) \end{Bmatrix} \exp(j\omega t), & (3) \\ i = \begin{Bmatrix} A\sqrt{\dfrac{C}{L}}\exp(-j\omega\sqrt{LC}x) \\ -B\sqrt{\dfrac{C}{L}}\exp(j\omega\sqrt{LC}x) \end{Bmatrix} \exp(j\omega t), & (4) \end{cases}$$

where A and B are constants. We can easily understand that the voltage in a DCC is composed of two members: the first one expresses the travelling wave $V_\mathrm{f} = A\exp(-j\omega\sqrt{LC}x)\exp(j\omega t)$ that goes forward; the second one expresses the reflected wave $V_\mathrm{r} = B\exp(j\omega\sqrt{LC}x)\exp(j\omega t)$ that reflects back. The current is composed similarly. The reflected wave tells us that the input voltage is reflected and back to the source, resulting in $v \neq v'$ (see Fig. 2). It also means that the power/signal transfer is inefficient and the source may be destroyed.

Let us consider an actual DCC with the whole impedance Z_0, as shown in Figure 3. The whole length is ℓ. R and G are omitted for simplicity. The input connects with a power source having a voltage v_p and an input impedance Z_p. The output connects with a load having an impedance Z_ℓ. The voltage reflection coefficient Γ is expressed by

$$|\Gamma| \equiv \frac{V_\mathrm{r}}{V_\mathrm{f}} = \left|\frac{Z_\ell - Z_0}{Z_\ell + Z_0}\right|. \qquad (5)$$

Equation 5 demonstrates that if the $|\Gamma|$ value becomes zero, no reflection is accomplished. In other words, every DCC impedance comprising a whole radio-frequency circuit must be the same. That is the *impedance matching*.

However, impedance matching is difficult to learn because it requires deep understanding of both electrical circuits and electromagnetism. Here we propose a step-by-step education program comprising four practices so that students learn it without difficulty. This program ranges over three grades in the Kosen system.

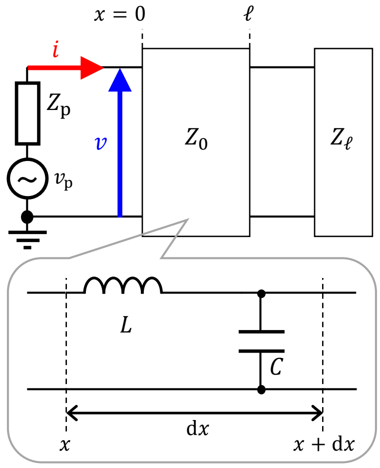

Figure 3. An actual DCC. The whole impedance Z_0, and the length is ℓ. R and G are omitted for simplicity. The input connects with a couple of power source with a voltage v_p and input impedance Z_p. The output connects with a load having an impedance Z_ℓ.

Practices 1&2: Electromagnetic Wave Theory and Audio Impedance Matching in KHz Region

First, in a half of the fourth grade, we educate students to electromagnetism with Maxwell's equations, which unifies electric, magnetic, and light. They learn the skin effect and electromagnetic waves. They learn that an AC voltage can be treated as a wave having a certain wavelength.

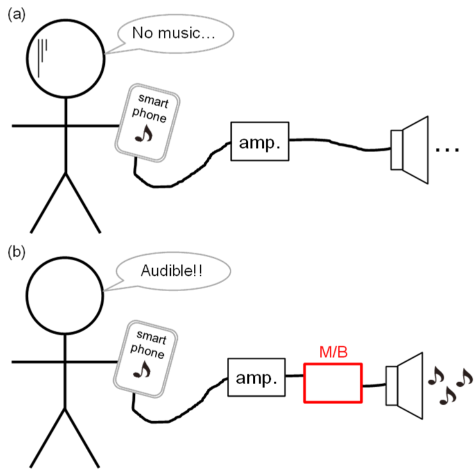

Figure 4. Music from a smart phone played through a speaker: (a) without and (b) with a M/B.

Based on this electromagnetism theory, in a quarter of the next fifth grade, we educate students to a familiar impedance matching example, an audio circuit. We can listen to any music from a smart phone. To open the music for everyone, we may connect an amplifier and a speaker to the smart phone; however, we hear the music very small or cannot as a result, as shown in Figure 4(a), because the output impedance of the smart phone or amplifier and that of the speaker are unmatched.

Thus, as shown in Figure 4(b), by inserting an impedance matching box (matching box, M/B) to the line, everyone can listen to the music. The M/B adjusts different impedances between connected components so that the reflected wave set to zero, explained later. However, in actual audio systems, several transformers instead of a M/B are used. This is because the audible range is very wide from 20 to 20,000 Hz, in contrast, the M/B can adjust impedance at only one frequency. Small impedance mismatching with a transformer at some frequency is adjusted by an equalizer.

These phenomena are electrically explained how efficiently we transfer power. Figure 5 shows an electrical circuit with a power source with a voltage V_0 and an output impedance $Z_0 = R_0 + jX_0$ and a load with an impedance $Z' = R' + jX'$. We need to make the power for the load P' the maximum.

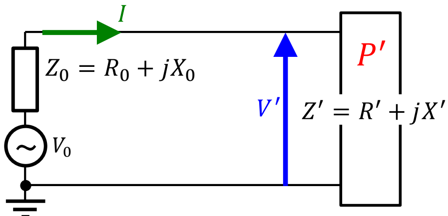

Figure 5. An electrical circuit to explain maximizing power supply to a load.

The complex power for the load $P_c{}'$ is expressed by

$$P_c{}' = V'\bar{I} = \frac{Z'}{Z_0 + Z'} V_0 \cdot \frac{\overline{V_0}}{\overline{Z_0 + Z'}}. \tag{6}$$

Thus, P' is obtained by

$$\therefore P' = \mathrm{Re}(P_c{}') = \frac{R'}{(R_0 + R')^2 + (X_0 + X')^2} {V_0}^2. \tag{7}$$

The P' takes the maximum when ${\partial P'}/_{\partial R'} = {\partial P'}/_{\partial X'} = 0$. A couple of $R' = R_0$ and $X' = -X_0$ satisfy the condition, leading $Z' = \overline{Z_0}$. Figure 6 shows the $P' - R'$ curve. The P' goes to the maximum 50% at $R' = R_0$. Especially, when $R' < R_0$, the value becomes smaller steeply. Thus, impedance matching is more important in this area. Because the output impedance of power sources is generally $R_0 = 50\ \Omega$, and that of actual speakers is $R' = 8\ \Omega$ or much less than $50\ \Omega$, the above matching condition is critical.

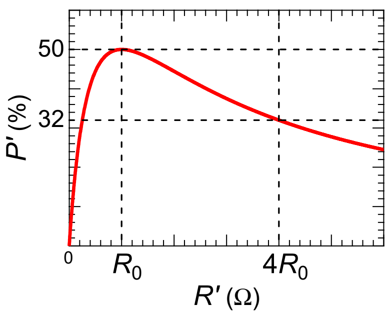

Figure 6. $P' - R'$ curve for impedance matching.

Figure 7 shows an electrical circuit with a M/B. The M/B comprises an inductor (coil) L and two variable capacitors, C_1 and C_2. The former capacitor set parallel to the load is called a load capacitor, and the latter series is a phase capacitor.

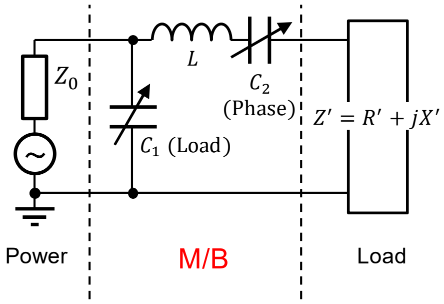

Figure 7. An electrical circuit with a M/B.

The values of these passive elements are determined by Equations 8–10. Eq. 10 comes from Eq. 9.

$$\begin{cases} C_1 = \dfrac{\sqrt{R'(R_0 - R')}}{\omega R' R_0} > 0 & (8) \\[2ex] C_2 = \dfrac{1}{\omega\left\{(\omega L + X') - \sqrt{R'(R_0 - R')}\right\}} > 0 & (9) \\[2ex] L > \dfrac{1}{\omega}\left\{\sqrt{R'(R_0 - R')} - X'\right\} & (10) \end{cases}$$

We provide its practice to students. First, we give each of their teams a target frequency for impedance matching of the circuit in Fig. 7. They start to measure impedance of the given speaker $Z' = R' + j\left(\omega L' - \frac{1}{\omega C'}\right)$. Then, they make a suitable coil by rotating a supporting tool like pedaling a bicycle, as shown in Figure 8.

Figure 8. A photograph of making a handmade coil.

When the coil fabrication is finished, they measure its impedance $Z_L = R_L + j\left(\omega L - \frac{1}{\omega C_L}\right)$, where R_L and C_L are parasitic resistance and capacitance, respectively. They must add these parasitic values to $Z'$and then create a new speaker impedance $Z'' = (R' + R_L) + j\left(\omega L' - \frac{1}{\omega C'} - \frac{1}{\omega C_L}\right)$. The R' and X' in Eqs. 8–10 are changed to $R' + R_L$ and $\omega L' - \frac{1}{\omega C'} - \frac{1}{\omega C_L}$, respectively. According to Eq. 10, the minimum value of L, L_{min}, can be calculated as $L_{\mathrm{min}} = \frac{1}{\omega}\left\{\sqrt{R''(R_0 - R'')} - X''\right\}$. If $L > L_{\mathrm{min}}$, the handmade coil is proven to be suitable for this purpose. After selecting appropriate C_1 and C_2 according to Eqs. 8 and 9, they finally check the combined impedance of the M/B and speaker, Z, to be $50 + j0\ [\Omega]$.

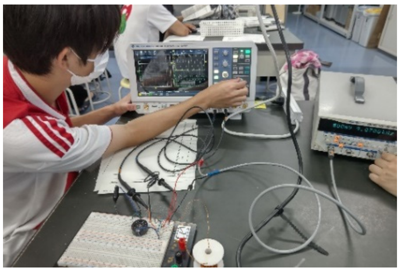

Figure 9. A photograph of impedance matching practice with an audio circuit.

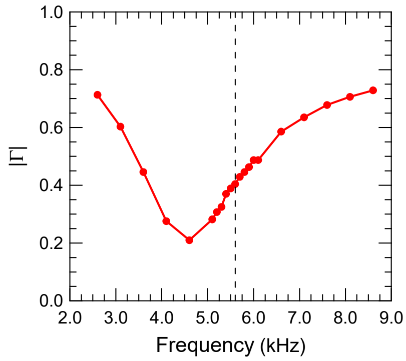

Figure 10. |Γ| value as a function of frequency. The target frequency is 5.6 kHz.

Students measure some parameters for impedance matching, as shown in Figure 9. A student team measured the absolute value of the voltage reflection coefficient, |Γ|, as a function of frequency (refer to Eq. 5), as shown in Figure 10. The target frequency was 5.6 kHz. The minimum value is obtained near the target frequency. The difference between the target and observed frequencies may be due to some line impedance.

The phase variation can be obtained by the Lissajous figures, as shown in Figure 11. Students measure the maximum height of the figure a and the difference between two points crossing the y-axis b. They finally obtain the phase angle θ by Equation 11. Note that this equation depends on inclination of the figure.

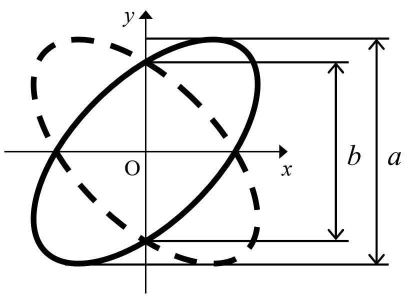

Figure 11. Lissajous figures.

$$\theta = \begin{cases} \sin^{-1}\dfrac{b}{a} & \text{(diagonal right, bold line),} \\ \pi - \left|\sin^{-1}\dfrac{b}{a}\right| & \text{(diagonal left, broken line).} \end{cases} \quad (11)$$

The team also measured θ as a function of frequency, as shown in Figure 12. The |Γ| presented in Fig. 10 is also shown as reference. The frequency where the minimum θ is obtained is the same value of the minimum |Γ|, demonstrating that $Z' = \overline{Z_0}$, as discussed before.

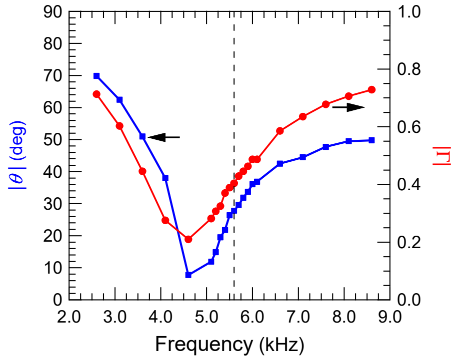

Figure 12. (left) θ and (right) |Γ| values as a function of frequency.

Practice 3: Impedance Matching in MHz Region with VNA

In a quarter of the sixth grade (first grade of the advanced course), students learn impedance matching in the MHz region. The two-port network model is used to analyze arbitrary electrical circuits. The Z- (impedance-) and Y- (admittance-) parameter analyses, as shown in Figure 13, are typical methods. The voltage and current input into a device under test (DUT) from the left side are V_1 and I_1, respectively. Those ones from the right side are V_2 and I_2, respectively. The relationship among these variables is expressed by Equations 12 and 13.

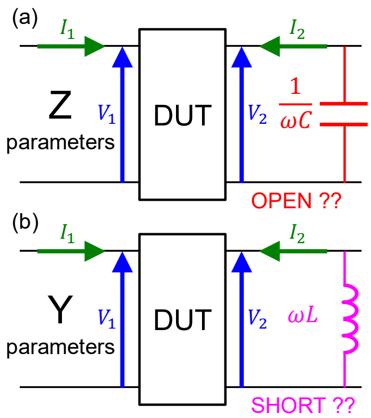

Figure 13. (a) Z- and (b) Y-parameter analyses.

$$\begin{cases} \text{Z:} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}, & (12) \\ \text{Y:} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}. & (13) \end{cases}$$

To obtain each Z-parameter from Eq. 12, the left or right hand current, I_1 or I_2, should be zero. In other words, an open condition must be established. However, at a radio frequency, because the capacitive reactance $X_C = {}^1/_{\omega C}$ goes to zero, the open condition becomes impossible. Similarly, to obtain each Y-parameter from Eq. 13, V_1 or V_2 should be zero. A short condition must be established. Any conductive lines accompany with inductance. Because the inductive reactance $X_L = \omega L$ goes to infinity at a radio frequency, the short condition becomes also impossible. After all, voltage and current cannot be measured independently at a radio frequency.

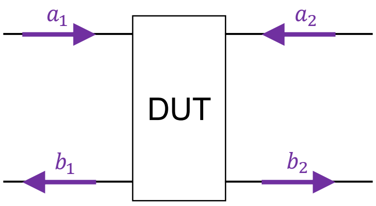

Figure 14. S-parameter analysis.

Instead of the Z- and Y-parameters, we must use S-parameters at a radio frequency. The input and output variables are the square roots of power a_n and b_n, respectively, as shown in Figure 14. The relationship is expressed by Equation 14,

$$\text{S:}\ \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}. \tag{14}$$

We obtain $\Gamma = S_{11}$. In the S-parameter analysis, the Smith chart, as shown in Figure 15, is convenient to understand the change of equivalent passive elements. From Eq. 5, we obtain Equation 15 that explains the Smith chart circle,

$$\left(u - \frac{R}{R+1}\right)^2 + v^2 = \left(\frac{1}{R+1}\right)^2, \tag{15}$$

where $Z \equiv {Z_\ell}/{Z_0} = R + jX$ and $\Gamma = u + jv$. On the Smith chart, resistance R increases from left to right (0 to ∞). Similarly, inductive reactance $+X$ increases clockwise, and capacitive reactance $-X$ increases anticlockwise. The $|\Gamma|$ equals to the line length (radius) between the measured point A and origin O, and the angle made by the line and positive x-axis indicates its phase angle θ. Impedance matching is accomplished at O, where $(R, X) = (1,0)$ and $(u, v) = (0,0)$. Using the Smith chart, we can know these values visibly without complicated calculation.

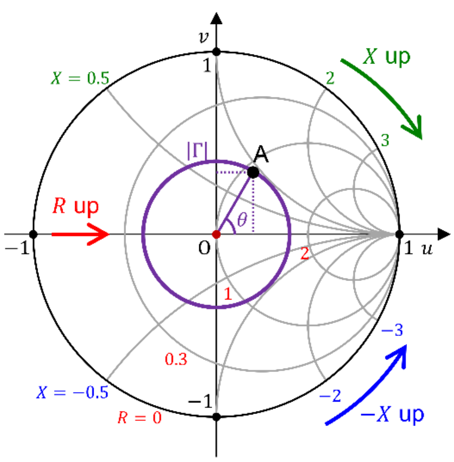

Figure 15. Smith chart.

We made a M/B used in the MHz region by using commercial elements: two trimmer capacitors and an inductor. The load is a simple LCR series. Figure 16 shows these ones.

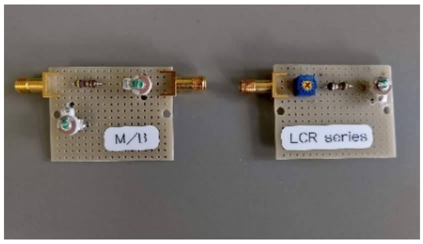

Figure 16. (left) M/B and (right) load for impedance matching in the MHz region.

The S-parameter analysis is performed with a vector network analyzer (VNA). The VNA is one of the most expensive apparatuses in electrical engineering. However, presently we can use a low-cost but high-performance VNA, NanoVNA. A NanoVNA is a palm-size apparatus. It can work either solely or with a PC. Students set their LCR loads arbitrarily to each other, and then they compete for the impedance matching. Figure 17 shows the practice.

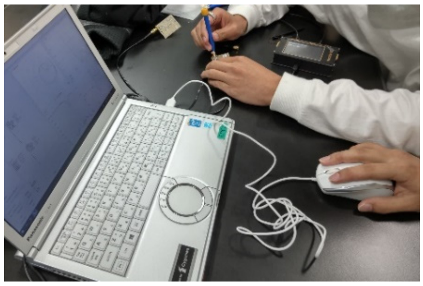

Figure 17. A photograph of impedance matching practice in the MHz region using a palm-size NanoVNA (top) with its control PC (left).

Practice 4: Impedance Matching of Waveguide in GHz Region with VNA

In the following quarter, students learn impedance matching in the GHz region, as a compilation of this education program. When the target frequency reaches GHz, the skin effect in electromagnetism becomes critical. The skin effect demonstrates that the current flows only at the surface of a conductor. The depth where the current can flow is estimated by the skin depth. The depth in copper at 2.45 GHz is approximately 1 μm. Thus, it is difficult to flow large current on a general circuit at such a frequency because the actual line resistance is so high.

To overcome this problem, the electrical power or signal should be flown as an electromagnetic wave in the space that is regulated by a waveguide. Impedance matching in a waveguide is basically to generate a standing wave inside the waveguide so that the short condition is accomplished at the walls. Figure 18 shows a waveguide system. The system is composed of a waveguide-to-coaxial adapter, a straight waveguide, and a dummy load having a sliding short bar to move the end wall. The latter two components are handmade.

The electromagnetic wave in a waveguide takes some propagation modes. The most basic one is the TE_{10} mode, as shown in Figure 19. The TE_{10} electromagnetic wave propagates along the wavenumber vector $\boldsymbol{k}$ by repeating reflections at the wall diagonally. The electric field vector $\boldsymbol{E}$ is along the y-axis, and the magnetic field vector $\boldsymbol{B}$ is on the xz-plane.

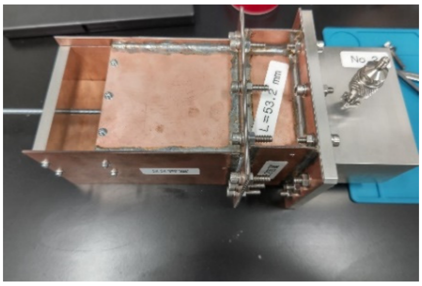

Figure 18. A waveguide system. The system is composed of (right) a waveguide-to-coaxial adapter, (center) a straight waveguide, and (left) a dummy load having a sliding short bar.

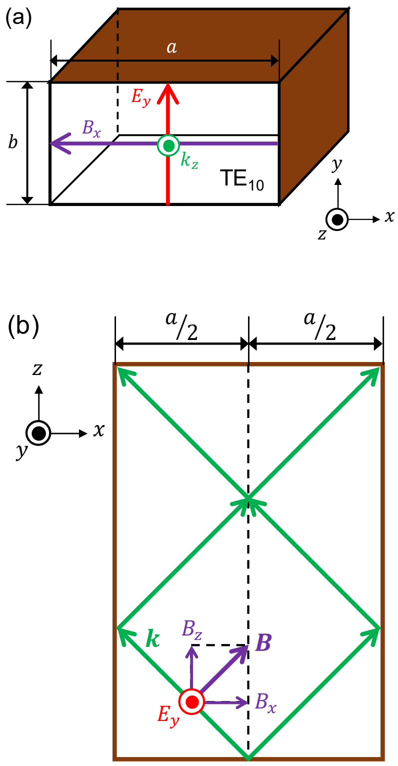

Figure 19. (a) Cross section and (b) top view of a TE_{10}-mode waveguide.

Students measure the S-parameters of a waveguide system using a NanoVNA, as shown in Figure 20. They learn variation of the characteristics by changing straight waveguide (from short one to the long) and moving the sliding short bar of the dummy load.

A student team measured $|\Gamma| = |S_{11}|$ as a function of frequency, as shown in Figure 21. The team obtained the minimum $|\Gamma|$ at 2.66 GHz in a waveguide of which the length is 246.4 mm. Like this, at present we have fortune that we can experience the VNA measurement easily with NanoVNAs.

Figure 20. A photograph of impedance matching practice of a waveguide (bottom) in the GHz region using a NanoVNA (right) and its control PC.

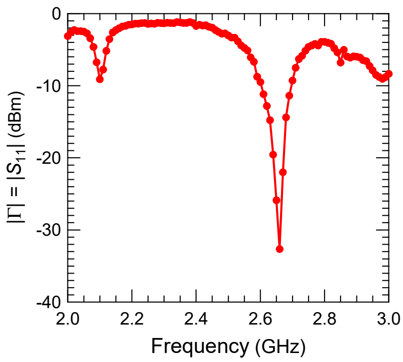

Figure 21. $|\Gamma| = |S_{11}|$ as a function of frequency (the LogMag characteristics in dBm).

Conclusions

A step-by-step education program of impedance matching in Ariake Kosen was explained. The education program comprises four practices. This education starts in a half of the fourth grade from electromagnetism theory with Maxwell's equations including the skin effect and electromagnetic wave (Practice 1). Actual impedance matching of an audio circuit in the KHz region is performed in a quarter of the next fifth grade (Practice 2). Finally, in the sixth grade (first grade of the advanced course), impedance matching of the circuit in the MHz region (Practice 3, a quarter) and that of a waveguide in the GHz region (Practice 4, another quarter) are experienced with the S-parameter analysis using a NanoVNA. Through this program, Kosen students will be expected to be radio-frequency electrical engineers.

Acknowledgements

The author (S. T.) acknowledges Mr. K. Ooi, formerly of ADTEC Plasma Technology Co., Ltd. for advising fabrication of waveguides and the NanoVNA-analysis.

References

C Pozar, D. M. (2011). Microwave Engineering. 4th ed. New York: Wiley.

NanoVNA V2 Official Site. https://nanorfe.com/nanovna-v2.html.